Mathematical simulation of the freezing time of water in small diameter pipes

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HIGHLIGHTS

• Mathematical simulation of the freezing of water in circular pipes.
• A one-dimensional, finite-length scale, transient heat conduction model approach was developed.
• Comparison of predictions of finite-length scale model and experiments were conducted.
• Model results compared well to experimental results.
• Application of the model to practical scenarios was considered and explored.

ABSTRACT

The cooling and freezing of stagnant water in cylindrical pipes were investigated under conditions of varying ambient air temperatures. Experiments were conducted to measure the transient temperature of water in a 50.8 mm nominal Schedule 80 steel pipe that was exposed to stagnant and forced air at temperatures ranging from −25 °C to −5 °C. There was a 10 vol.% air gap in the pipe to avoid excessive increase of fluid pressure during the freezing process. A one-dimensional transient heat conduction mathematical model was developed to estimate the freezing time of the liquid in the pipe. The model was based on the separation of variables method for a finite length-scale solidification problem, and was verified with the experimental transient temperature data. It was found that the model predicted the solidification time of the water in the pipe to within 15% of that which was measured by experiments for most of the tests that were conducted. The model was applied to predict the cooling and freezing behaviors of water in steel and copper pipes of various inner diameters and in insulated pipes. The results of the model and their agreement with experimental data suggest that a separation of variables method for a finite length-scale heat conduction problem may be applied to moving or free boundary problems to predict phase change phenomena.

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1. Introduction

The phenomenon of solidification of stagnant water in cylindrical pipes has received attention from numerous investigators given the detrimental effects of freezing such as the possible blockage and bursting of the pipes. When pipes are exposed to low-temperature environments, the water will experience cooling, supercooling of the liquid phase below the fusion temperature, solidification at the fusion temperature, and further cooling of the solid ice [1]. Gilpin [2] has shown that supercooled water in pipes is characterized by the growth of thin plate-like solid crystals known as dendritic ice, which forms in advance of the annular solid ice. The consequence of the formation of either dendritic ice or solid annular ice in the pipe is the possibility of flow blockage and bursting of the pipe due to the generation of large internal pressures. It has been shown that dendritic ice formation may be responsible for early blockage of the water pipes under certain conditions [2]. Sugawara et al. [3] and Oiwake et al. [4] conducted both numerical analyses and experimental studies to quantify the changes in internal pipe pressure during the freezing process, while Gordon [1] presented experimental results of pressures required to cause bursting of a variety of pipes used in building applications. The numerical analyses of Sugawara et al. [3] and Oiwake et al. [4] and the experimental work of Gordon [1] included the case where the pressure increase in the freezing fluid depressed the fusion point. They admitted that the continuous freezing point depression...
rendered analysis of the problem difficult, forcing the use of incremental time steps and numerical code.

Further experimental studies on the freezing of stagnant water in pipes have explored the effects of internal free convective motion during the solidification process. Due to the decrease in the density of water as it freezes [5], complicated flow patterns may emerge, which may have an effect on the rate of formation of dendritic and solid annular ice [6–8]. Gilpin [8] has shown specifically that for quiescent water with supercooling less than 2 °C, free convection produces non-uniform ice distribution. For greater amounts of supercooling, relatively uniform ice distribution is formed since the velocity of the advancing dendritic ice front exceeds that of the free convective motion.

The modeling of solidification phase change problems has been the subject of extensive research investigations [9–16]. Initial model studies focused on this phase change in slabs with semi-infinite extent [9] or in liquids that were initially at the fusion temperature [9,10], with the intent of finding the temperature distribution in the material during phase change and the location of the moving boundary. The non-linearity of the interface energy equation made analytical solutions of the temperature distribution difficult to obtain. This has resulted in computer-aided numerical solutions of the problem, and, as outlined in an early review by Muehlbauer and Sunderland [11], the use of several mathematical techniques such as the variational technique, the heat-balance integral technique, and the Riemann–Mellin contour integral technique. More recently, model studies have focused on the determination of the time required for freezing of materials. Pham [12] has extended Planck’s equation [13] to include sensible heat effects during the freezing of food-based materials with simple shapes. Other investigators such as Cleland et al. [14] and Hung [15] used numerical methods, coupled with experimental data, to predict the freezing time of irregularly shaped bodies of food. While these studies on the prediction of freezing time through the use of either simple models or numerical techniques are useful for solid food material, they do not enable the determination of the transient temperature distribution in the material or estimation of the transient solidification front during freezing.

The objectives of this study were to: (1) develop an analytical heat conduction model to estimate the freezing time of liquids in pipes; (2) conduct experiments to validate the model; and (3) extend the use of the model to cases of practical interest.

2. Experimental method

The freezing of water in cylindrical pipes was used as a basis to gather data for validation of the mathematical model that was developed in this study. In order to mitigate the risk of brittle fractures, all piping material and components were rated for low temperature service. A 50.8 mm (2 in) Schedule 80 carbon steel pipe (ASTM A333 Grade 6) that was approximately 305 mm (12 in) long was used for all the tests and is shown in Fig. 1. Thread-o-lets were welded at each end of the pipe as well as one in the middle to create a closed system and to allow insertion of instrumentation for temperature measurements. The total system volume was approximately 550 mL (18.6 fl oz).

A thermowell with a T-type thermocouple (PP-T-20, Omega, Montréal, QC, Canada) was inserted along the centerline of the pipe (ASTM A333 Grade 6) that was approximately 305 mm (12 in) long was used for all the tests and is shown in Fig. 1. Thread-o-lets were welded at each end of the pipe as well as one in the middle to create a closed system and to allow insertion of instrumentation for temperature measurements. The total system volume was approximately 550 mL (18.6 fl oz).

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pipe through one of the thread-o-lets and was used to measure the transient temperature of the water and ice during the cooling and freezing events. Thermal paste (8610 Non Silicone Heat Transfer Compound, M.G. Chemicals, Burlington, ON, Canada) was applied to the thermocouple junction to reduce contact resistance between the thermocouple and the thermowell. The thermowell had an outer diameter of 12.7 mm (0.50 in) and extended inside the pipe approximately 127 mm (5 in) (see Fig. 1). The other two orifices were plugged using 19 mm (0.75 in) NPT stainless steel plugs and Teflon tape. The ambient temperature within the freezer was measured using a T-type thermocouple. The temperature inside the duct was recorded using a J-type rigid probe-style thermocouple (TJ36-ICSS-14U-6-CC, Omega, Montréal, QC, Canada). Tap water was used as the process fluid. For all the tests, a 10 vol.% air gap was introduced into the pipe to avoid excessive increase of fluid pressure during the freezing process. The transient temperature data was collected with a data acquisition system (SCXI-1600, National Instruments, Austin, TX, USA) and a measurement and automation explorer software for data processing (National Instruments, Austin, TX, USA) was used. The data sampling interval was 1 Hz (1 sample per second). Once recorded, the data was then exported into a Microsoft Excel file for further analysis.

All the cooling and freezing tests were conducted in an 18.2 m$^3$ (640 ft$^3$) cold room freezer (Foster Refrigerator USA, Kinderhook, NY, USA). Five tests in which the ambient air was held stagnant at temperatures of $-5 \degree C$, $-10 \degree C$, $-15 \degree C$, $-20 \degree C$, and $-25 \degree C$ were conducted in a closed duct to reduce the effects of circulating air currents external to the pipe. Several tests were also conducted to simulate forced air cooling and freezing, over a temperature range of $-25$ to $-5 \degree C$, in increments of $5 \degree C$. In order to control the flow of air over the pipe, an experimental assembly was constructed of a 254 mm x 305 mm (10 in x 12 in) galvanized sheet metal duct, a 0.25 kW (0.33 hp) direct-drive tube-axial fan (DDA-12-10033B, Leader Fan Industries, Toronto, ON, Canada), a galvanized sheet metal 305 mm (12 in) round to 254 mm x 305 mm (10 in x 12 in) rectangular reducer, and a 41 mm (1.625 in) slotted C-channel tubing (McMaster-Carr, Aurora, USA) (see Fig. 2a). The overall dimensions of the assembly were 2 m x 0.66 m x 0.48 m (80 in x 25.9 in x 18.9 in) as shown in Fig. 2b. A triangular hole was cut into the duct to allow access for the test pipe. The entire assembly was elevated at approximately 30° to ensure that the air gap in the pipe remained at one end of the pipe. A digital manometer (475-1-FM, Dwyer Instruments, Michigan City, USA) was used to measure the air speed after exit from the fan and within the vicinity of the pipe.

Fig. 2. a) Components and b) dimensions of the forced air test apparatus.
3. Mathematical models

3.1. Cooling time

Cooling of the liquid in a freezer with ambient temperature, $T_\infty$, from an initial temperature ($T_i$) to its melting point, $T_m$, will occur before the initiation of freezing. The time required to cool the liquid from its initial temperature to the freezing point can be determined by considering a one-dimensional transient heat conduction model in cylindrical co-ordinates for the cooling of the stagnant fluid in the pipe. Assuming constant properties, the governing equation is

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha_l} \frac{\partial T}{\partial t}, \quad 0 \leq r \leq R_{\text{inn}}. \quad (1)$$

The boundary and initial conditions are,

$$\frac{\partial T(0, t)}{\partial r} = 0, \quad (2)$$

$$-k_l \frac{\partial T(R_{\text{inn}}, t)}{\partial r} = U[T(R_{\text{inn}}, t) - T_\infty], \quad (3)$$

$$T(r, 0) = T_i - T_\infty. \quad (4)$$

Ozisik [17] has solved Eq. (1) with the boundary and initial conditions of Eqs. (2)–(4) to give

$$T(r, t) = T_\infty + \frac{2u(T_i - T_\infty)}{R_{\text{inn}}} \sum_{n=1}^{\infty} \frac{J_0(\beta_n r)}{J_0(\beta_n R_{\text{inn}})} \exp(-\alpha_l \beta_n^2 t), \quad (5)$$

where $u = U/r_k$, and the eigenvalues are

$$\beta_n = \left( \frac{n \pi}{R_{\text{inn}}} \right), \quad (6)$$

The overall heat transfer coefficient ($U$) during this single-phase heat transfer problem can be estimated by utilizing correlation equations that are available from the work of others. The pipe material, insulation, and outside heat transfer coefficient ($h_o$) constituted the overall heat transfer coefficient at the outer surface of the pipe. In general

$$\frac{1}{D} = R' + \frac{1}{h_o, \text{ convection}} + \frac{1}{h_o, \text{ radiation}}. \quad (7)$$

The thermal resistance of the pipe material and any insulation is given as

$$R' = \frac{K_{\text{pipe}}}{D_{\text{pipe}}} + \frac{K_{\text{insul}}}{D_{\text{insul}}} = \frac{1}{2k_{\text{pipe}}D_{\text{pipe}}} + \frac{1}{2k_{\text{insul}}D_{\text{insul}}}. \quad (8)$$

Free convection and radiation will occur at the outer surface of the pipe during the stagnant air tests. The orientation of the pipes in this study will be horizontal. The free convective heat transfer coefficient for horizontal pipes is given by Churchill and Chu [18] as

$$h_{\text{conv}} = \frac{k_{\text{air}}}{D_o} \left[ 0.6 + \frac{0.387 \text{Ra}^{1/6}}{[1 + \left( \frac{0.559 \text{Pr}^{1/3}}{10} \right)^{9/16}]} \right]^2 \cdot \text{Ra}_{D_o} \leq 10^{12}. \quad (9)$$

Eq. (9) is valid for laminar free convective flow over cylinders within an infinite fluid medium for uniform surface temperature or uniform heat flux. In this study, the tests were conducted in a closed channel in which the cylinder occupied approximately 25% of the height of the channel. It is possible that free convective motion occurred within the gap between the channel wall and the cylinder, increasing the heat transfer coefficient beyond the value predicted by Eq. (9). It is also possible that with high values of Rayleigh number, the free convection motion would be turbulent within the gap, further increasing the heat transfer coefficient. There are no correlation equations for the heat transfer coefficients for this specific case; however, verification of the Rayleigh number would confirm the possible occurrence of free convective currents. Ostrach [19] has shown that the critical Rayleigh number that is required to induce free convective motion in enclosed spaces bounded by rigid planes is 1708. Turbulence will first appear at a Rayleigh number that is approximately 5830. The length scale in the Rayleigh number is the gap distance between the planes. Under stagnant air conditions, the approximate radiation heat transfer coefficient is [20]

$$h_{\text{rad}} = \frac{\epsilon_{\text{pipe}} \sigma (T_\infty^4 - T_\infty^4)}{T_\infty - T_\infty}. \quad (10)$$

where the Stefan–Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ and $T$ is in Kelvin.

Due to the difficulties in estimating the overall heat transfer coefficient for the stagnant air tests, the models developed in this study, coupled with the experimental results, will be used to estimate values of the overall heat transfer coefficients. Mcquiston et al. [21] has also suggested that the determination of overall heat transfer coefficients over bare pipes exposed to free convection is complicated due to the lack of knowledge of the transient surface temperature of the pipe upon which the heat transfer coefficients will depend. Experimental data for the overall heat transfer coefficients for horizontal bare steel pipes that were presented by Mcquiston et al. [21] will also be used in this study.

Forced convection will dominate the heat transfer at the outer surface of the pipe during the forced air tests. Perkins and Leppert [22] have shown that the average heat transfer coefficient over the pipe will be affected by flow in both the laminar boundary layer region and in the separated region over the pipe, with significant increases in the local heat transfer coefficient in the separated region of flow. Provided that the Reynolds number is lower than the critical value for laminar-to-turbulent transition for flow over circular bodies ($2 \times 10^5$), the correlation equation for the average heat transfer coefficient is [22]

$$h_{\text{conv}} = \frac{k_{\text{air}}}{D_o} \left[ 0.31 \text{Re}_{D_o}^{0.50} + 0.11 \text{Re}_{D_o}^{0.67} \text{Pr}^{0.40} \right]. \quad (11)$$

Perkins and Leppert [22] accounted for the variation of dynamic viscosity across the boundary layer by using a ratio of the dynamic viscosity at the cylinder wall to the bulk dynamic viscosity. The ratio was assumed to be unity for this study since the variation of dynamic viscosity of gases with temperature is small.

In this study, the cooling and freezing experiments were conducted in a closed rectangular duct system (see Fig. 2). It has been shown that when the diameter of a pipe occupies a portion of the cross-sectional area of a ducted channel, blockage of the gas flow will occur [23]. Blockage will affect the pressure and velocity distributions around the cylindrical pipe, which will in turn affect the location of the separation point on the cylindrical pipe and the average heat transfer coefficient [22]. To that end, Robinson et al. [23] proposed an empirically determined blockage correction...
The properties of the air in external thermal condition of the pipes that were used in this present study. Therefore, Eq. (11) becomes

\[ T_{\text{convection}} = \frac{k_{\text{air}}}{D_o} \left( 1 + \frac{D_o}{H} \right) \left[ 0.31\text{Re}_{D_o}^{0.50} + 0.11\text{Re}_{D_o}^{0.67} \right] \rho_l \theta^{0.40}, \]

(12)

where \( H \) is the height of the channel, and it is 254 mm (10 in) in this study (see Fig. 2b).

The use of the blockage correction factor that was developed from experimental work by Robinson et al. [23] for cylinders with non-isothermal surfaces will most likely relate closely to the true thermal condition of the pipes that were used in this present study. The properties of the air in external flow over the pipe were determined at the film temperature, \( T_{\text{film}} \). The surface temperature of the pipe was assumed to be the average of the initial temperature and the fusion temperature of the water in the pipe.

3.2. Freezing time

A one-dimensional transient heat conduction model, in cylindrical co-ordinates, was used to estimate the freezing time of the liquid in the pipes. Fig. 3 shows a schematic of the model used in the analysis. The pipe was of finite length and was filled with quiescent liquid at the freezing point. In order to induce solidification, the surrounding ambient air temperature was less than the freezing point of the liquid. It was assumed that the growth of the solid layer was axisymmetric and that all properties were constant.

The governing equations of the temperature distribution in the solid and liquid phases are

\[ T_s(r, t) = T_f, \quad 0 \leq r < r_{\text{in}}(t), \]

(13)

\[ 1 \frac{\partial}{\partial r} \left( r \frac{\partial T_s}{\partial r} \right) = \frac{1}{\alpha_s} \frac{\partial T_s}{\partial t}, \quad r_{\text{in}}(t) \leq r \leq R_{\text{inn}}. \]

(14)

The boundary and initial conditions are

\[ T_s(r_{\text{in}}, t) = T_f, \]

(15)

\[ \frac{-k_s \frac{\partial T_s(R_{\text{inn}}, t)}{\partial r}}{\frac{\partial T_s}{\partial r}} = U[T_f(R_{\text{inn}}, t) - T_{\infty}], \]

(16)

\[ T_s(r, 0) = T_f, \]

(17)

\[ r_{\text{in}}(0) = R_{\text{inn}}. \]

(18)

Conservation of energy at the solid–liquid interface will yield another boundary condition, the interface energy equation, which establishes the problem as a free boundary problem, with \( r_{\text{in}}(t) \) representing the transient location of the interface. The solid–liquid interface energy equation is

\[ k_s \frac{\partial T_s(r_{\text{in}}, t)}{\partial r} = \rho_l \frac{dT_{\text{in}}}{dt}. \]

(19)

The initial condition of Eq. (17) suggests that the axisymmetric solid forms instantaneously at \( t = 0 \). This assumption serves to simplify the mathematical development of the model. However, Gilpin [2] has shown that ice will form initially as plate-like dendritic crystals that are distributed throughout the water and will be eventually engulfed by solid as the freezing event proceeds.

This is a finite length-scale problem in which the pipe is bounded at a radius of \( r = R_{\text{inn}} \) (see Fig. 3). The governing equation of the temperature distribution in the solid phase (Eq. (14)) was solved by using the separation of variables method [17]. Given the non-homogeneity of Eqs. (15) and (16), the solution for \( T_s(r, t) \) was assumed to be the sum of two functions, one \( \Psi \) depending on \( r \) and \( \tau \) for the homogeneous solution, and the other, \( \phi \), on \( r \) only, for the particular solution, so that

\[ T_s(r, t) = \Psi(r, t) + \phi(r). \]

(20)

Eq. (20) was substituted into Eq. (14), and after separation gives

\[ 1 \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) = \frac{1}{\alpha_s} \frac{\partial \Psi}{\partial t}, \]

(21)

\[ 1 \frac{d}{dr} \left( r \frac{d\phi}{dr} \right) = 0. \]

(22)

Substitution of Eq. (20) into the boundary and initial conditions of Eqs. (15)–(17), followed by separation will produce homogeneous boundary conditions for \( \Psi \) from Eqs. (15) and (16). Since \( \phi \) is a function of \( r \) only, Eq. (17) will remain non-homogeneous after substitution of Eq. (20).

The solution for \( \Psi(r, t) \) was assumed to be the product of two functions, with \( \Psi \) depending on \( r \) only, and \( \tau \), on \( t \) only, so that

\[ \Psi(r, t) = \mathcal{R}(r)\tau(t). \]

(23)

Eq. (23) was substituted into Eq. (21), and after separation

\[ \frac{d^2 \mathcal{R}_k}{dr^2} + \frac{1}{r} \frac{d\mathcal{R}_k}{dr} + \lambda_k^2 \mathcal{R}_k = 0. \]

(24)

\[ \frac{d\tau_k}{dr} + \alpha_s \lambda_k^2 \tau_k = 0. \]

(25)

Integration of Eqs. (24) and (25) gives

\[ \mathcal{R}_k(r) = A_k J_0(\lambda_k r) + B_k Y_0(\lambda_k r), \]

(26)

\[ \tau_k(t) = C_k \exp\left(-\alpha_s \lambda_k^2 t\right), \]

(27)

where \( A_k, B_k, \) and \( C_k \) are integration constants.
Application of the boundary condition at \( r = r_{in} \) (Eq. (15)) for \( \Psi(r_{in}, t) = 0 \) to Eq. (26) gives
\[
B_k = -A_k \frac{J_0(\lambda_k)Y_1(\lambda_k R_{in})}{Y_0(\lambda_k R_{in})}.
\]  
(28)

The boundary condition of Eq. (16) at \( r = R_{in} \) for \(-k_\rho(\partial \Psi(R_{in}, t))/\partial r = U \Psi(R_{in}, t)\) is applied to Eq. (26) in order to find an expression for the eigenvalue, \( \lambda_k \)
\[
\left[ \frac{J_1(\lambda_k R_{in}) - \frac{k_\rho L_{in}}{\rho C_{p,s}} Y_1(\lambda_k R_{in})}{J_0(\lambda_k R_{in})} - \frac{k_\rho L_{in}}{\rho C_{p,s}} \right] = \frac{U}{k_\rho} \lambda_k.
\]  
(29)

One of the values of the separation constant is \( \lambda_k = \lambda_0 = 0 \). Substitution of \( \lambda_k = 0 \) into Eqs. (24) and (25) and application of the boundary conditions of Eqs. (15) and (16) to the solution of Eq. (24) showed that \( \Psi_0(r) = 0 \) and \( \psi_0(r, t) = R_0(r_0, t) = 0 \). Therefore
\[
\psi(r, t) = \sum_{k=1}^{\infty} a_k \left[ \frac{J_0(\lambda_k r) - J_0(\lambda_k R_{in})}{J_0(\lambda_k R_{in})} Y_0(\lambda_k r) \right] \exp \left( -\alpha_k^2 \frac{t}{t} \right),
\]  
(30)

where \( a_k = A_k C_k \).

The solution of the particular problem that is governed by Eq. (22) is
\[
\phi(r) = E \ln r + F,
\]  
(31)

where \( E \) and \( F \) are constants of integration.

Application of the boundary conditions at \( r = r_{in} \) (Eq. (15)) for \( \phi(r_{in}) = T_i \) and \( r = R_{in} \) (Eq. (16)) for \(-k_\rho(d\phi(R_{in})/dr) = U \phi(R_{in}) - T_i \) to Eq. (31) gives
\[
F = T_i - \frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} \ln \frac{r_{in}}{R_{in}},
\]  
(32)

\[
E = \frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} - \frac{k_\rho L_{in}}{\rho C_{p,s}} \ln r_{in}.
\]  
(33)

Finally, the temperature distribution in the solid phase is
\[
T_s(r, t) = \frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} \ln \frac{r}{r_{in}} + \sum_{k=1}^{\infty} a_k \left[ \frac{J_0(\lambda_k r) - J_0(\lambda_k R_{in})}{J_0(\lambda_k R_{in})} \right] Y_0(\lambda_k r) \exp \left( -\alpha_k^2 \frac{t}{t} \right).
\]  
(34)

Eq. (24) is a Sturm–Liouville equation and the boundary conditions of Eqs. (15) and (16) are homogeneous, after transformation in terms of \( \Psi \). Therefore, orthogonality can be applied to determine \( a_k \) [17]. The initial condition of Eq. (17) was used to produce
\[
\frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} \ln \frac{r}{r_{in}} + \sum_{k=1}^{\infty} a_k \left[ \frac{J_0(\lambda_k r) - J_0(\lambda_k R_{in})}{J_0(\lambda_k R_{in})} \right] Y_0(\lambda_k r) = 0.
\]  
(35)

It is uncertain if the \([J_0(\lambda_k r) - (J_0(\lambda_k R_{in}))/Y_0(\lambda_k r)]]Y_0(\lambda_k r)\) term in Eq. (35) is orthogonal. Therefore, orthogonality cannot be applied implicitly to find \( a_k \). However, a numerical check was conducted for \( \lambda_k \), which showed that only one of the values of \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \) or \( \lambda_6 \) were such that the \([J_0(\lambda_k r') - (J_0(\lambda_k R_{in}))/Y_0(\lambda_k r')]]Y_0(\lambda_k r')\) term was very large for either \( k = 1, 2, \ldots, \) or 6, compared to other values of \( k \). It is important to note that any one of the values could be used for a given solution since only one value of \( k \) will produce a relatively large value of the \([J_0(\lambda_k r) - (J_0(\lambda_k R_{in}))/Y_0(\lambda_k r)]]Y_0(\lambda_k r')\) term. So, one value of \( a_k \) will apply. Therefore, the integral of each term in Eq. (35) over the region \( r_{in} < r < R_{in} \) of the solid was determined to give
\[
\frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} \ln \frac{r}{r_{in}} + \sum_{k=1}^{\infty} a_k \left[ \frac{J_0(\lambda_k r) - J_0(\lambda_k R_{in})}{J_0(\lambda_k R_{in})} \right] Y_0(\lambda_k r) \exp \left( -\alpha_k^2 \frac{t}{t} \right) = 0.
\]  
(36)

Eq. (36) will yield approximate values of \( a_k \). In addition, the expression was solved numerically since there are no closed form analytical solutions of the integrals of the Bessel functions of zero order, \( J_0(\lambda_k r) \) and \( Y_0(\lambda_k r) \).

Knowledge of the transient location of the solid–liquid interface, \( r_{in}(t) \), during the freezing event will enable estimation of the liquid freezing time within a pipe of inner radius, \( R_{in} \). Differentiation of Eq. (34) at \( r = r_{in} \) and substitution into the solid–liquid interface energy equation of Eq. (19) produced
\[
k_\rho \left[ \frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} \ln \frac{r_{in}}{R_{in}} - \frac{k_\rho L_{in}}{\rho C_{p,s}} \right] + \sum_{k=1}^{\infty} a_k \lambda_k \left[ \frac{J_0(\lambda_k R_{in})}{J_0(\lambda_k R_{in})} \right] \left[ J_1(\lambda_k r_{in}) - \frac{J_0(\lambda_k R_{in})}{Y_0(\lambda_k R_{in})} \right] Y_0(\lambda_k r_{in}) \exp \left( -\alpha_k^2 \frac{t}{t} \right) \right.
\]
\[+ \left. \sum_{k=1}^{\infty} a_k \frac{k_\rho L_{in}}{\rho C_{p,s}} \right] \exp \left( -\alpha_k^2 \frac{t}{t} \right) = \rho_L L_{in} \left[ \frac{U(T_i - T_\infty)}{U \ln \left( \frac{r_{in}}{R_{in}} \right)} \ln \frac{r_{in}}{R_{in}} - \frac{k_\rho L_{in}}{\rho C_{p,s}} \right].
\]  
(37)

The ice thickness as a function of time is
\[
\chi(t) = R_{in} - r_{in}(t).
\]  
(38)

A MATLAB (MathWorks, Inc., Natick, MA, USA) code was used to solve the expressions for the first 6 values of \( \lambda_k \) [Eq. (29)]. By discretizing the spatial domain, values of \( a_k \) in Eq. (36) were calculated. The values of time that corresponded to each \( r_{in} \) value were calculated using Eq. (37). The differential equation in Eq. (37) was estimated by a first-order finite difference approximation. The infinite series summation in Eq. (37) was found to be insignificant, with a dominant first term on the order of approximately \( 10^{-70} \). This simplified the expression to
\[
\frac{k_\rho U(T_i - T_\infty)}{\rho L \ln \left( \frac{r_{in}}{R_{in}} \right)} \frac{dr_{in}}{dt}.
\]  
(39)

Eq. (39) gives a solution for the time corresponding to the equally-spaced discretized ice thickness, \( \chi \). A water–ice system was

<table>
<thead>
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<th>Table 1: General properties of solid ice [5].</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Melting point, ( T_m )</td>
</tr>
<tr>
<td>Thermal diffusivity, ( a_\rho )</td>
</tr>
<tr>
<td>Specific heat capacity, ( c_{p,s} )</td>
</tr>
<tr>
<td>Thermal conductivity, ( k_s )</td>
</tr>
<tr>
<td>Density, ( \rho_s )</td>
</tr>
<tr>
<td>Latent heat of fusion, ( L )</td>
</tr>
</tbody>
</table>
investigated in this study. General properties of solid ice that were used in the model are shown in Table 1 [5].

4. Results and discussion

4.1. Temperature traces

Transient temperature traces were generated for the various conditions of ambient air temperature for both the stagnant and forced air tests of cooling and freezing of water. Fig. 4 shows the transient temperature traces when the average ambient air temperature was approximately $-14\,^\circ C$ to $-16\,^\circ C$, where the set-point temperature was $-15\,^\circ C$. The ambient air temperature fluctuated due to cycling of the fans in the cold room freezer. The traces show a region of liquid cooling from approximately $21\,^\circ C$, where the set-point temperature was $-15\,^\circ C$. The ambient air temperature fluctuated due to cycling of the fans in the cold room freezer. The traces show a region of liquid cooling from approximately $21\,^\circ C$, a distinct solidification plateau, and after complete solidification, further cooling of the solid ice. The temperature profile of Fig. 4a for the stagnant air test shows evidence of supercooling of the liquid with the amount of supercooling on the order of $2.3\,^\circ C$. During supercooling, dendritic ice growth occurs. As shown in the figure, at $-2.3\,^\circ C$, the water temperature returns to approximately $0\,^\circ C$, at which point, an annulus of solid ice begins to form at the inner wall surface of the pipe.

An increase in the cooling rate (change in temperature per unit time) by exposing the liquid and pipe to forced ambient air produced a transient temperature profile that was devoid of evidence of significant supercooling (see Fig. 4b). This is expected since increased cooling rates would enable rapid development of the solid annular ice in the pipe. The solid ice would quickly engulf the thin layer of dendritic ice growth, significantly reducing the amount of supercooling. Tables 2 and 3 show the average air temperatures, the initial water temperatures, supercooled water temperatures, and the approximate cooling rates that were obtained for the stagnant and forced air tests in this study. As the cooling rate increased due to a decrease in the ambient air temperature (see Table 2) or the use of forced air (see Table 3), the amount of supercooling decreased. The standard deviation was presented with the average ambient temperature ($T_{\infty}$). The number of data points that were used to generate this average was in excess of 5000.

The increase in the cooling rate of the water during the forced air tests was due primarily to the forced convection heat transfer at the outer surface of the pipe. The speed of the free-stream air flow was measured to be $31\,\text{km/h}$ ($19.3\,\text{miles/h}$). This free-stream air velocity was chosen since the American Society of Heating, Refrigeration, and Air-conditioning Engineers (ASHRAE) have suggested that outdoor wind speeds in the winter will typically be about $24\,\text{km/h}$ ($15\,\text{miles/h}$) [24]. In the forced air tests of this study, the velocity around the pipes would be higher due to blockage in the duct channel. With the blockage correction factor from Robinson et al. [23] that is shown in Eq. (12) and data on the outer pipe diameter and duct height, the air velocity over the pipe may range from 56 to $68\,\text{km/h}$ ($35-43\,\text{miles/h}$) as the flow moves from laminar to laminar-separated flow over the cylindrical pipe.

![Fig. 4. Transient temperature traces of water in 50.8 mm nominal diameter Schedule 80 steel pipes under a) stagnant and b) force air tests.](image)

<table>
<thead>
<tr>
<th>$T_{\infty}$, $^\circ C$</th>
<th>$T_i$, $^\circ C$</th>
<th>$T_{super}$, $^\circ C$</th>
<th>Cooling rate, $^\circ C/s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5.3 \pm 1.2$</td>
<td>$21.6$</td>
<td>$-3.2$</td>
<td>$1.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>$-9.1 \pm 1.2$</td>
<td>$21.9$</td>
<td>$-2.5$</td>
<td>$3.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$-13.7 \pm 0.9$</td>
<td>$21.0$</td>
<td>$-2.3$</td>
<td>$3.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>$-16.1 \pm 3.5$</td>
<td>$21.3$</td>
<td>$-2.2$</td>
<td>$5.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$-22.2 \pm 0.7$</td>
<td>$21.4$</td>
<td>$-1.8$</td>
<td>$6.3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T_{\infty}$, $^\circ C$</th>
<th>$T_i$, $^\circ C$</th>
<th>$T_{super}$, $^\circ C$</th>
<th>Cooling rate, $^\circ C/s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-6.4 \pm 0.9$</td>
<td>$21.0$</td>
<td>$-1.7$</td>
<td>$1.4 \times 10^{-2}$</td>
</tr>
<tr>
<td>$-11.6 \pm 0.8$</td>
<td>$19.1$</td>
<td>$-1.3$</td>
<td>$1.7 \times 10^{-2}$</td>
</tr>
<tr>
<td>$-16.0 \pm 0.7$</td>
<td>$22.4$</td>
<td>$0.1$</td>
<td>$2.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$-21.2 \pm 0.6$</td>
<td>$21.2$</td>
<td>$0.1$</td>
<td>$2.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>$-26.0 \pm 0.6$</td>
<td>$19.7$</td>
<td>$0.1$</td>
<td>$1.8 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

The cooling and freezing times — predicting the experimental results

The cooling and freezing times of liquids in pipes will depend on the transient temperature distributions of the liquid and solid phases. The temperature distribution models of Eqs. (5) and (34) were used to estimate the cooling and freezing times, respectively. The overall heat transfer coefficient ($U$) is a major parameter that needs to be specified in order to determine the temperature distributions and the cooling and freezing times. Table 4 shows the values of the heat transfer coefficients that were used to estimate the cooling and freezing times. The overall heat transfer coefficient for the stagnant air tests with free convection was determined by fitting the experimental data results for the cooling of water in an ambient at $-5.3\,^\circ C$ and freezing at $-6.6\,^\circ C$. It was assumed that the overall heat transfer coefficient of $15.4\,\text{W m}^{-2}\,\text{K}^{-1}$ was constant for all the other stagnant air tests in the study. For free convection of air over a 20 mm outer diameter horizontal cylinder, Özisik has
suggested that the heat transfer coefficient be on the order of $10^1 \text{W m}^{-2} \text{K}^{-1}$ [17]. Mcquiston et al. [21] have also shown that for 50.8 mm diameter horizontal bare steel pipes, the combined effects of free convection and radiation would produce an overall heat transfer coefficient on the order of $13 \text{W m}^{-2} \text{K}^{-1}$. The predicted value of the overall heat transfer coefficient in this study for the stagnant air falls within the order-of-magnitude that was found by other investigators [17,21].

Eqs. (7) and (12) were used to estimate the overall heat transfer coefficient for the forced air tests. Due to the forced convective nature of the tests, the contributions of radiation and the thermal resistance of the metal pipe wall to the overall heat transfer coefficient was small. As expected, the overall heat transfer coefficients in the forced air tests were nearly an order-of-magnitude greater than those of the stagnant air tests. Eqs. (11) and (12) will be valid for the case of laminar flow over the pipe, when the Reynolds number is lower than the critical Reynolds number for laminar-to-turbulent transition for flow over cylindrical bodies ($2 \times 10^5$). The Reynolds number will be based on the undisturbed, free-stream air velocity.

In this study, and for a free-stream air velocity of 31 km/h, the Reynolds number was approximately $4.0 \times 10^4$, which is less than the critical Reynolds number, indicating laminar flow. Small variations in the Reynolds numbers occurred due to variations in the film temperature. In industrial practice, ASHRAE have suggested outdoor wind speeds of 24 km/h [24]. At this speed, the flow would become turbulent if the pipe outer diameter, including the thickness of any insulation, was approximately 390 mm (15.4 in) or larger.

The cooling and freezing times, as determined from Eqs. (5) and (34), respectively are shown in Fig. 5 for the stagnant tests at various ambient air temperatures. In all the tests in this study, a thermowell with a 12.7 mm outer diameter was inserted in the center of the pipe. Therefore, the cooling time was determined at the outer surface of the thermowell, where $r = 18.3 \text{mm}$ and the freezing time was determined as the time required for ice growth from the inner wall of the pipe to the outer surface of the thermowell. The cooling time of the water in the pipe was taken to be the time required for the water temperature to decrease from the initial temperature, $T_i$ to the supercooled temperature, $T_{super}$. Fig. 5 shows that as the ambient air temperature increases, the cooling and freezing times increase. Deviations between the experimentally measured cooling times and those predicted by the model could be as high as 25% (see Fig. 5a). This is most likely due to the assumptions that were made during estimation of the heat transfer coefficient. For free convection, coupled with radiation, as is the case in this study, the heat transfer coefficient will depend on the surface temperature of the pipe, which changes during the cooling process and will vary for different ambient air temperatures. However, it was assumed that the heat transfer coefficient was constant over the range of ambient air temperatures.

### Table 4

Overall heat transfer coefficients, $U$.

<table>
<thead>
<tr>
<th>Temperature, $T_{\infty}$, $^\circ\text{C}$</th>
<th>Stagnant air</th>
<th>Forced air</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$, $\text{W m}^{-2} \text{K}^{-1}$</td>
<td>$U$, $\text{W m}^{-2} \text{K}^{-1}$</td>
</tr>
<tr>
<td>-6.2</td>
<td>15.4</td>
<td>-6.4</td>
</tr>
<tr>
<td>-10.5</td>
<td>15.4</td>
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<td>-16.2</td>
</tr>
<tr>
<td>-18.6</td>
<td>15.4</td>
<td>-21.2</td>
</tr>
<tr>
<td>-23.3</td>
<td>15.4</td>
<td>-26.1</td>
</tr>
</tbody>
</table>

![Fig. 5. Plots of a) cooling time, b) freezing time, and c) total time versus temperature of stagnant ambient air for water in 50.8 mm nominal diameter Schedule 80 steel pipes.](image)
constant during each test. Further, Fig. 4 shows that during cooling of the liquid, the ambient air temperature is not constant, but decreased to the desired set-point temperature. The model of Eq. (5) assumed a constant ambient temperature, which will produce deviations in the model predictions and the experimental results for cooling time. During freezing of the water in the pipe, it is most likely that the surface temperature of the pipe was close to the melting point of water (0 °C). Therefore, any variation in the heat transfer coefficient would be due primarily to changes in temperature-dependent properties. In addition, the ambient temperature, though it fluctuated slightly during the freezing period of the water, significant decreases were not observed (see Fig. 4). Fig. 5b shows that the maximum deviation between the experimentally determined freezing time and the model prediction is no more than 13% for any ambient temperature that was used in the study. The total time required to cool and freeze the water in the pipes is shown in Fig. 5c. There is good agreement between the predictions of the models and the experiments for the total time, with a maximum deviation of no more than 7%.

The novelty of this study is such that a separation of variables method is used to provide estimates of the freezing time in a moving or free boundary problem. Other investigators have developed alternate models to predict the freezing time of materials such as foodstuffs. Pham [12] has modified Planck’s equation [13] for predicting freezing times by incorporating the sensible heat effects into their model. To that end, Fig. 5b shows the predictions of freezing times from the model developed by Pham [12]. The figure shows that the model over-predicts the freezing times by approximately 25–45%, supporting the need for a new model to refine the predictions of freezing time.

The models for the estimation of the cooling and freezing times were applied to the case where forced air was used to drive the cooling and freezing processes of the water in the pipe. Fig. 6 shows that the cooling and freezing times are significantly reduced when forced air cooling is used, in lieu of free convection and radiation with stagnant air (see Fig. 5). The deviations in the liquid cooling time between the model predictions and the experimental results can be as high as 50%, as shown in Fig. 6a. These deviations decrease as the ambient air temperature decreases. The deviations are also lower for the freezing times (see Fig. 6b), and are less than 8% when the total of the cooling and freezing times are considered (see Fig. 6c). The large deviations observed for the prediction of the liquid cooling times may be attributed to the non-constant ambient air temperatures at the beginning of the forced air tests.

4.3. Cooling and freezing – practical scenarios

The cooling and freezing of liquids in pipes are of particular importance in applications that involve heating, ventilating, and air-conditioning (HVAC) or the transport of liquids outdoor in industrial applications. When the fluids become quiescent in the pipes that are exposed to cold ambient air conditions, the stagnant fluids will cool and freeze more rapidly than if they were flowing through the pipe. For the case of bare, uninsulated horizontal pipes, Mcquiston et al. [21] have provided data on the overall heat transfer coefficients during free convection and radiation for pipes with outer diameters of 12.7 mm (0.5 in) to 127 mm (5 in), where the temperature difference between the surrounding air and the pipe surface varies from \( \Delta T = 28 \, ^\circ C (50 \, ^\circ F) \) to \( \Delta T = 278 \, ^\circ C (500 \, ^\circ F) \). Fig. 7 presents a graphical summary of the maximum overall heat

![Fig. 6. Plots of a) cooling time, b) freezing time, and c) total time versus temperature of forced ambient air for water in 50.8 mm nominal diameter Schedule 80 steel pipes.](image)
transfer coefficients that were estimated from the data from McQuiston et al. [21] for pipes filled with liquid at an initial temperature of 20 °C and exposed to ambient air temperatures from –5 °C to –25 °C. The surface of the pipe was assumed to be 20 °C to produce temperature differences that ranged from ΔT = 25 °C to ΔT = 45 °C. Overall heat transfer coefficients obtained at temperature differences below 28 °C were approximate since no data was provided by McQuiston et al. [21]. The figure shows that as the pipe size increased and the temperature difference decreased, the overall heat transfer coefficient decreased. The values reported in Fig. 7 represent the maximum overall heat transfer coefficients since as the fluid cools, the surface temperature of the pipe will also decrease, reducing the temperature difference and the overall heat transfer coefficient.

Fig. 8 shows curves of the model prediction of the total time required for the cooling and freezing of water in completely filled metal pipes of various inner diameters. The results of Fig. 8a for bare, uninsulated pipes show that for pipes with inner diameters less than 40 mm, the total times required for cooling and freezing of the water were close when the stagnant ambient air temperature is –15 °C or less. The curves nearly coalesce for smaller pipe inner diameters at low stagnant ambient air temperatures. Similar to the model results of Figs. 5a and 6a, the cooling time of the water in the pipe was taken to be the time required for the water temperature to decrease from the initial temperature (20 °C) to the supercooled temperature (∼2 °C). The supercooled temperature of −2 °C was chosen based on the results observed in Table 2, where the average supercooled temperature was –2.4 °C. In practical scenarios, there will be no thermowell inserted in the center of the pipe. Therefore, the cooling time was determined at the pipe center and the freezing time was determined as the time required for ice growth from the inner wall of the pipe to the center of the pipe. Given the high thermal conductivity of metal pipes, their resistance to heat transfer was neglected. This will enable the use of the curves of Fig. 8 with a variety of metal pipes, including copper and steel pipes. The model results presented in Fig. 8 were generated under the assumption that the internal pressure in the pipe is relieved as the water froze. It is possible that in practical applications, as the water freezes, the pipe may be blocked, causing an increase in the internal pressure in the pipe through the freezing event. The Clapeyron–Clausius equation will show that as the internal pressure on the water increases, the fusion temperature of the water will decrease. Therefore, no distinct solidification plateau, similar to that shown in Fig. 4, will be present. The effects of internal pressurization will form the basis of future work beyond this study.

Insulation may be used to increase the total time required for the cooling and freezing of the water in the pipe. There are a variety of insulation materials that are available on the market for use in industrial and HVAC applications. However, ASHRAE Standard 90.1-2013 [25] for energy efficiency in buildings specifies that for pipes with less than 100 mm nominal diameters that contain hot water between 41 °C and 60 °C or cold water between 4 °C and 16 °C, the insulation thickness should be between 15 and 40 mm and have a thermal conductivity of 0.030–0.040 W m⁻¹ K⁻¹. For this study, it was assumed that a 25 mm thick insulation is used to protect the pipes thermally, and the thermal conductivity of the insulation is 0.035 W m⁻¹ K⁻¹. Eq. (8) shows the expression to calculate the resistance due to insulation Rinsul for cylindrical pipes and tubes. Table 5 presents typical values of the inner and outer diameters of standard tubing and pipe insulation based on the pipe outer diameter for a 25 mm nominal insulation thickness [24]. The values obtained from Eq. (8) for the resistance due to insulation will be used in Eq. (7) to estimate the overall heat transfer coefficient. The heat transfer coefficient due to the combination of free convection and radiation can be obtained from Fig. 7. The total times required to cool and freeze water in insulated pipes are shown in Fig. 8b. The figure shows clearly that the use of insulation increases the total time required to cool and freeze the water by nearly an order of magnitude when compared to bare, uninsulated pipes. While this observation is intuitive, it has been observed experimentally by other investigators [1]. The extended times due to the use of insulation will reduce the cooling rates of the water in the pipes.

![Fig. 7. Maximum overall heat transfer coefficients for bare horizontal metal pipes in stagnant ambient air [21].](image7.png)

![Fig. 8. Model prediction of the total (cooling + freezing) time in stagnant ambient air for water in a) bare and b) insulated pipes as functions of pipe inner diameter.](image8.png)
in most cases, they will be insulated. Exposure to winter winds at bare, uninsulated pipes. The amount of supercooling may be larger than that observed in dendritic ice that is formed. It is expected that in insulated pipes, cooling rates will increase supercooling and the volume fraction of ice. Model prediction of the total cooling and freezing times of water in a) bare and insulated 50.8 mm nominal Schedule 80 steel pipes exposed to cold air with a speed of 24 km/h. The outer diameter of the steel pipe is 60.3 mm, and as per ASHRAE [24], for a 25 mm thick insulation, the inner and outer diameters of the insulation are 61 mm and 114 mm, respectively. The figure shows a similar trend to that observed in Fig. 8, in which the insulation significantly increased the total time required for cooling and freezing of water in the pipe.

5. Conclusions

This study focused on the estimation of the cooling and freezing time of water in cylindrical pipes that were exposed to stagnant and forced air at temperatures that ranged from −25 °C to −5 °C, in increments of 5 °C. A model that was based on the separation of variables method was developed and used to estimate the freezing time of water in both bare and insulated pipes. The model was validated with experimental transient temperature data of water during the cooling and freezing processes in a bare 50.8 mm nominal Schedule 80 steel pipe.

The estimates of the cooling and freezing times of the water that were obtained from the model were in good agreement with the experimental measurements. Larger deviations were observed in the cooling times due to the assumption of constant ambient air temperature in the cooling phase in the model, while in the experiments, the ambient temperature decreased from an initial value to the set-point temperature at the initial stage of that phase. A further source of discrepancy in the forced air test was likely due to uncertainties in the estimates of the heat transfer coefficient external to pipe.

Future work and extensions of this study and model are possible. In particular, the effects of internal pressurization on the freezing time could be modeled analytically. This extension of the model could be coupled with pressure vessel analysis to estimate the burst pressures to predict failure in piping where freezing may occur. Other liquid material systems could also be explored. Water is a unique material in that its density decreases as it experiences a transition from liquid to solid. Materials such as oils and non-Newtonian fluids may pose other unique issues as they cool, freeze, and are pressurized in the cylindrical pipes.

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References


