Physics 107 HOMEWORK ASSIGNMENT #6

Cutnell & Johnson, 7th edition

Chapter 6: Problems 10, 24, 30, 42, 62

*10  A 55-kg box is being pushed a distance of 7.0 m across the floor by a force \( \vec{P} \) whose magnitude is 150 N. The force \( \vec{P} \) is parallel to the displacement of the box. The coefficient of kinetic friction is 0.25. Determine the work done on the box by each of the four forces that act on the box. Be sure to include the proper plus or minus sign for the work done by each force.

**24  Multiple-Concept Example 5 reviews many of the concepts that play a role in this problem. An extreme skier, starting from rest, coasts down a mountain slope that makes an angle of 25.0° with the horizontal. The coefficient of kinetic friction between her skis and the snow is 0.200. She coasts down a distance of 10.4 m before coming to the edge of a cliff. Without slowing down, she skis off the cliff and lands downhill at a point whose vertical distance is 3.50 m below the edge. How fast is she going just before she lands?

30  A 55.0-kg skateboarder starts out with a speed of 1.80 m/s. He does +80.0 J of work on himself by pushing with his feet against the ground. In addition, friction does −265 J of work on him. In both cases, the forces doing the work are nonconservative. The final speed of the skateboarder is 6.00 m/s. (a) Calculate the change \( \Delta PE = PE_f - PE_0 \) in the gravitational potential energy. (b) How much has the vertical height of the skater changed, and is the skater above or below the starting point?

*42  A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of \( r = 36 \text{ m} \). Neglect friction and air resistance. What must be the height \( h \) of the first hill so that the skier just loses contact with the snow at the crest of the second hill?

**62  A 1900-kg car experiences a combined force of air resistance and friction that has the same magnitude whether the car goes up or down a hill at 27 m/s. Going up a hill, the car’s engine needs to produce 47 hp more power to sustain the constant velocity than it does going down the same hill. At what angle is the hill inclined above the horizontal?
10. **REASONING AND SOLUTION** The applied force does work

\[ W_p = Ps \cos 0^\circ = (150 \text{ N})(7.0 \text{ m}) = 1.0 \times 10^3 \text{ J} \]

The frictional force does work

\[ W_f = f_k s \cos 180^\circ = -\mu_k F_N s \]

where \( F_N = mg \), so

\[ W_f = -(0.25)(55 \text{ kg})(9.80 \text{ m/s}^2)(7.0 \text{ m}) = -940 \text{ J} \]

The normal force and gravity do no work, since they both act at a 90° angle to the displacement.

24. **REASONING** It is useful to divide this problem into two parts. The first part involves the skier moving on the snow. We can use the work-energy theorem to find her speed when she comes to the edge of the cliff. In the second part she leaves the snow and falls freely toward the ground. We can again employ the work-energy theorem to find her speed just before she lands.

**SOLUTION** The drawing at the right shows the three forces that act on the skier as she glides on the snow. The forces are: her weight \( mg \), the normal force \( F_N \), and the kinetic frictional force \( f_k \). Her displacement is labeled as \( s \). The work-energy theorem, Equation 6.3, is

\[ W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

where \( W \) is the work done by the net external force that acts on the skier. The work done by each force is given by Equation 6.1,

\[ W = (F \cos \theta)s \]

so the work-energy theorem becomes

\[ \frac{(mg \cos 65.0^\circ)s + (f_k \cos 180^\circ)s + (F_N \cos 90^\circ)s}{W} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

Since \( \cos 90^\circ = 0 \), the third term on the left side can be eliminated. The magnitude \( f_k \) of the kinetic frictional force is given by Equation 4.8 as \( f_k = \mu_k F_N \). The magnitude \( F_N \) of the normal force can be determined by noting that the skier does not leave the surface of the slope, so \( a_y = 0 \text{ m/s}^2 \). Thus, we have that \( \Sigma F_y = 0 \), so
The magnitude of the kinetic frictional force becomes \( f_k = \mu_k F_N = \mu_k mg \cos 25.0^\circ \). Substituting this result into the work-energy theorem, we find that

\[
\frac{(mg \cos 65.0^\circ) s + (\mu_k mg \cos 25.0^\circ) (\cos 180^\circ) s}{W} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2
\]

Algebraically eliminating the mass \( m \) of the skier from every term, setting \( \cos 180^\circ = -1 \) and \( v_0 = 0 \) m/s, and solving for the final speed \( v_f \) gives

\[
v_f = \sqrt{2gs(\cos 65.0^\circ - \mu_k \cos 25.0^\circ)}
\]

\[
= \sqrt{2(9.80 \text{ m/s}^2)(10.4 \text{ m})[\cos 65.0^\circ - (0.200) \cos 25.0^\circ]} = 7.01 \text{ m/s}
\]

The drawing at the right shows her displacement \( s \) during free fall. Note that the displacement is a vector that starts where she leaves the slope and ends where she touches the ground. The only force acting on her during the free fall is her weight \( mg \). The work-energy theorem, Equation 6.3, is

\[
W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2
\]

The work \( W \) is that done by her weight, so the work-energy theorem becomes

\[
\frac{(mg \cos \theta) s}{W} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2
\]

In this expression \( \theta \) is the angle between her weight (which points vertically downward) and her displacement. Note from the drawing that \( s \cos \theta = 3.50 \text{ m} \). Algebraically eliminating the mass \( m \) of the skier from every term in the equation above and solving for the final speed \( v_f \) gives

\[
v_f = \sqrt{v_0^2 + 2g(s \cos \theta)}
\]

\[
= \sqrt{(7.01 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(3.50 \text{ m})} = 10.9 \text{ m/s}
\]
30. **REASONING AND SOLUTION**

a. The work done by non-conservative forces is given by Equation 6.7b as

\[ W_{nc} = \Delta KE + \Delta PE \quad \text{so} \quad \Delta PE = W_{nc} - \Delta KE \]

Now

\[ \Delta KE = \frac{1}{2} mv_i^2 - \frac{1}{2} mv_0^2 = \frac{1}{2} (55.0 \text{ kg}) [(6.00 \text{ m/s})^2 - (1.80 \text{ m/s})^2] = 901 \text{ J} \]

and

\[ \Delta PE = 80.0 \text{ J} - 265 \text{ J} - 901 \text{ J} = -1086 \text{ J} \]

b. \( \Delta PE = mg (h - h_0) \) so

\[ h - h_0 = \frac{\Delta PE}{mg} = \frac{-1086 \text{ J}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = -2.01 \text{ m} \]

Thus, the skater’s vertical position has changed by 2.01 m, and the skater is below the starting point.

42. **REASONING AND SOLUTION** If air resistance is ignored, the only nonconservative force that acts on the skier is the normal force exerted on the skier by the snow. Since this force is always perpendicular to the direction of the displacement, the work done by the normal force is zero. We can conclude, therefore, that mechanical energy is conserved.

\[ \frac{1}{2} mv_0^2 + mgh_0 = \frac{1}{2} mv_f^2 + mgh_f \]

Since the skier starts from rest \( v_0 = 0 \text{ m/s} \). Let \( h_f \) define the zero level for heights, then the final gravitational potential energy is zero. This gives

\[ mgh_0 = \frac{1}{2} mv_f^2 \quad (1) \]

At the crest of the second hill, the two forces that act on the skier are the normal force and the weight of the skier. The resultant of these two forces provides the necessary centripetal force to keep the skier moving along the circular arc of the hill. When the skier just loses contact with the snow, the normal force is zero and the weight of the skier must provide the necessary centripetal force.

\[ mg = \frac{mv_f^2}{r} \quad \text{so that} \quad v_f^2 = gr \quad (2) \]
Substituting this expression for $v_f^2$ into Equation (1) gives

$$mgh_0 = \frac{1}{2} mgr$$

Solving for $h_0$ gives

$$h_0 = r \frac{36 \text{ m}}{2} = 18 \text{ m}$$

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62. **REASONING AND SOLUTION** The following drawings show the free-body diagrams for the car in going both up and down the hill. The force $F_R$ is the combined force of air resistance and friction, and the forces $F_U$ and $F_D$ are the forces supplied by the engine in going uphill and downhill respectively.

![Free-body diagram going up the hill](image)

![Free-body diagram going down the hill](image)

Writing Newton's second law in the direction of motion for the car as it goes uphill, taking uphill as the positive direction, we have

$$F_U - F_R - mg \sin \theta = ma = 0$$

Solving for $F_U$, we have

$$F_U = F_R + mg \sin \theta$$

Similarly, when the car is going downhill, Newton's second law in the direction of motion gives

$$F_R - F_D - mg \sin \theta = ma = 0$$

so that

$$F_D = F_R - mg \sin \theta$$

Since the car needs 47 hp more to sustain the constant uphill velocity than the constant downhill velocity, we can write

$$P_U = P_D + \Delta P$$
where $\bar{P}_U$ is the power needed to sustain the constant uphill velocity, $\bar{P}_D$ is the power needed to sustain the constant downhill velocity, and $\Delta P = 47 \text{ hp}$. In terms of SI units,

$$\Delta P = (47 \text{ hp}) \left( \frac{746 \text{ W}}{1 \text{ hp}} \right) = 3.51 \times 10^4 \text{ W}$$

Using Equation 6.11 ($\bar{P} = F \bar{v}$), the equation $\bar{P}_U = \bar{P}_D + \Delta P$ can be written as

$$F_U \bar{v} = F_D \bar{v} + \Delta P$$

Using the expressions for $F_U$ and $F_D$, we have

$$(F_R + mg \sin \theta) \bar{v} = (F_R - mg \sin \theta) \bar{v} + \Delta P$$

Solving for $\theta$, we find

$$\theta = \sin^{-1} \left( \frac{\Delta P}{2mg\bar{v}} \right) = \sin^{-1} \left( \frac{3.51 \times 10^4 \text{ W}}{2(1900 \text{ kg})(9.80 \text{ m/s}^2)(27 \text{ m/s})} \right) = 2.0^\circ$$