Chapter 18

**15 Interactive Solution** 18.15 provides a model for solving this type of problem. Two small objects, A and B, are fixed in place and separated by 3.00 cm in a vacuum. Object A has a charge of +2.00 \( \mu \text{C} \), and object B has a charge of −2.00 \( \mu \text{C} \). How many electrons must be removed from A and put onto B to make the electrostatic force that acts on each object an attractive force whose magnitude is 68.0 N?

*21* An electrically neutral model airplane is flying in a horizontal circle on a 3.0-m guideline, which is nearly parallel to the ground. The line breaks when the kinetic energy of the plane is 50.0 J. Reconsider the same situation, except that now there is a point charge of +\( q \) on the plane and a point charge of −\( q \) at the other end of the guideline. In this case, the line breaks when the kinetic energy of the plane is 51.8 J. Find the magnitude of the charges.

**23** ssm A small spherical insulator of mass 8.00 \( \times 10^{-2} \) kg and charge +0.600 \( \mu \text{C} \) is hung by a thin wire of negligible mass. A charge of -0.900 \( \mu \text{C} \) is held 0.150 m away from the sphere and directly to the right of it, so the wire makes an angle \( \theta \) with the vertical (see the drawing). Find (a) the angle \( \theta \) and (b) the tension in the wire.

*39* ssm A rectangle has a length of 2\( d \) and a height of \( d \). Each of the following three charges is located at a corner of the rectangle: +\( q_1 \) (upper left corner), +\( q_2 \) (lower right corner), and −\( q \) (lower left corner). The net electric field at the (empty) upper right corner is zero. Find the magnitudes of \( q_1 \) and \( q_2 \). Express your answers in terms of \( q \).

**76** Concept Questions The drawing shows a positive point charge +\( q_1 \), a second point charge \( q_2 \) that may be positive or negative, and a spot labeled \( P \), all on the same straight line. The distance \( d \) between the two charges is the same as the distance between \( q_1 \) and the point \( P \). With \( q_2 \) present, the magnitude of the net electric field at \( P \) is twice what it is when \( q_1 \) is present alone. (a) When the second charge is positive, is its magnitude smaller than, equal to, or greater than the magnitude of \( q_1 \)? Explain your reasoning. (b) When the second charge is negative, is its magnitude smaller than, equal to, or greater than that in question (a)? Account for your answer.

Problem Given that \( q_1 = +0.50 \mu \text{C} \), determine \( q_2 \) when it is (a) positive and (b) negative. Verify that your answers are consistent with your answers to the Concept Questions.
Chapter 19

**9** ssm The potential at location $A$ is 452 V. A positively charged particle is released there from rest and arrives at location $B$ with a speed $v_B$. The potential at location $C$ is 791 V, and when released from rest from this spot, the particle arrives at $B$ with twice the speed it previously had, or $2v_B$. Find the potential at $B$.

**10** A particle is uncharged and is thrown vertically upward from ground level with a speed of 25.0 m/s. As a result, it attains a maximum height $h$. The particle is then given a positive charge $+q$ and reaches the same maximum height $h$ when thrown vertically upward with a speed of 30.0 m/s. The electric potential at the height $h$ exceeds the electric potential at ground level. Finally, the particle is given a negative charge $-q$. Ignoring air resistance, determine the speed with which the negatively charged particle must be thrown vertically upward, so that it attains exactly the maximum height $h$. In all three situations, be sure to include the effect of gravity.

**24** A positive charge $+q_1$ is located to the left of a negative charge $-q_2$. On a line passing through the two charges, there are two places where the total potential is zero. The first place is between the charges and is 4.00 cm to the left of the negative charge. The second place is 7.00 cm to the right of the negative charge. (a) What is the distance between the charges? (b) Find $q_1/q_2$, the ratio of the magnitudes of the charges.

**25** Charges $q_1$ and $q_2$ are fixed in place, $q_2$ being located at a distance $d$ to the right of $q_1$. A third charge $q_3$ is then fixed to the line joining $q_1$ and $q_2$ at a distance $d$ to the right of $q_2$. The third charge is chosen so the potential energy of the group is zero; that is, the potential energy has the same value as that of the three charges when they are widely separated. Determine the value for $q_3$, assuming that (a) $q_1 = q_2 = q$ and (b) $q_1 = q$ and $q_2 = -q$. Express your answers in terms of $q$.

**26** One particle has a mass of $3.00 \times 10^{-3}$ kg and a charge of $+8.00 \mu$C. A second particle has a mass of $6.00 \times 10^{-3}$ kg and the same charge. The two particles are initially held in place and then released. The particles fly apart, and when the separation between them is 0.100 m, the speed of the $3.00 \times 10^{-3}$ kg particle is 125 m/s. Find the initial separation between the particles.
Chapter 18

15. **REASONING** The electrons transferred increase the magnitudes of the positive and negative charges from 2.00 $\mu$C to a greater value. We can calculate the number $N$ of electrons by dividing the change in the magnitude of the charges by the magnitude $e$ of the charge on an electron. The greater charge that exists after the transfer can be obtained from Coulomb’s law and the value given for the magnitude of the electrostatic force.

**SOLUTION** The number $N$ of electrons transferred is

$$N = \frac{|q_{\text{after}}| - |q_{\text{before}}|}{e}$$

where $|q_{\text{after}}|$ and $|q_{\text{before}}|$ are the magnitudes of the charges after and before the transfer of electrons occurs. To obtain $|q_{\text{after}}|$, we apply Coulomb’s law with a value of 68.0 N for the electrostatic force:

$$F = k \frac{|q_{\text{after}}|^2}{r^2} \quad \text{or} \quad |q_{\text{after}}| = \sqrt{\frac{Fr^2}{k}}$$

Using this result in the expression for $N$, we find that

$$N = \frac{\sqrt{\frac{Fr^2}{k}} - |q_{\text{before}}|}{e} = \frac{\sqrt{(68.0 \text{ N})(0.0300 \text{ m})^2} - 2.00 \times 10^{-6} \text{ C}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 3.8 \times 10^{12}$$

21. **REASONING** This is a problem that deals with motion in a circle of radius $r$. As Chapter 5 discusses, a centripetal force acts on the plane to keep it on its circular path. The centripetal force $F_c$ is the name given to the net force that acts on the plane in the radial direction and points toward the center of the circle. When there are no electric charges present, only the tension in the guideline supplies this force, and it has a value $T_{\text{max}}$ at the moment the line breaks. However, when there is a charge of $+q$ on the plane and a charge of $-q$ on the guideline at the center of the circle, there are two contributions to the centripetal force. One is the electrostatic force of attraction between the charges and, since the charges have the same magnitude, its magnitude $F$ is given by Coulomb’s law (Equation 18.1) as $F = k \frac{|q|^2}{r^2}$. The other is the tension $T_{\text{max}}$ which is characteristic of the rope and has the same value as when no charges are present. Whether or not charges are present, the centripetal force is equal to the mass $m$ times the centripetal acceleration, according to Newton’s second law and stated in Equation 5.3, $F_c = mv^2/r$. In this expression $v$ is the
speed of the plane. Since we are given information about the plane’s kinetic energy, we will use the definition of kinetic energy, which is \( KE = \frac{mv^2}{2} \), according to Equation 6.2.

**SOLUTION** From the definition of kinetic energy, we see that \( mv^2 = 2(KE) \), so that Equation 5.3 for the centripetal force becomes

\[
F_c = \frac{mv^2}{r} = \frac{2(KE)}{r}
\]

Applying this result to the situations with and without the charges, we get

\[
T_{\text{max}} + \frac{k|q|^2}{r^2} = \frac{2(KE)_{\text{charged}}}{r} \quad (1) \quad \quad \quad T_{\text{max}} = \frac{2(KE)_{\text{uncharged}}}{r} \quad (2)
\]

Subtracting Equation (2) from Equation (1) eliminates \( T_{\text{max}} \) and gives

\[
\frac{k|q|^2}{r^2} = \frac{2[(KE)_{\text{charged}} - (KE)_{\text{uncharged}}]}{r}
\]

Solving for \(|q|\) gives

\[
|q| = \sqrt{\frac{2r[(KE)_{\text{charged}} - (KE)_{\text{uncharged}}]}{k}} = \frac{2(3.0 \text{ m})(51.8 \text{ J} - 50.0 \text{ J})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2} = 3.5 \times 10^{-5} \text{ C}
\]

23. SSM **REASONING** The charged insulator experiences an electric force due to the presence of the charged sphere shown in the drawing in the text. The forces acting on the insulator are the downward force of gravity (i.e., its weight, \( W = mg \)), the electrostatic force \( F = k|q_1||q_2|/r^2 \) (see Coulomb's law, Equation 18.1) pulling to the right, and the tension \( T \) in the wire pulling up and to the left at an angle \( \theta \) with respect to the vertical as shown in the drawing in the problem statement. We can analyze the forces to determine the desired quantities \( \theta \) and \( T \).

**SOLUTION.**

a. We can see from the diagram given with the problem statement that

\[
T_x = F \quad \text{which gives} \quad T \sin \theta = k|q_1||q_2|/r^2
\]

and

\[
T_y = W \quad \text{which gives} \quad T \cos \theta = mg
\]
Dividing the first equation by the second yields

$$\frac{T \sin \theta}{T \cos \theta} = \tan \theta = \frac{k |q_1| |q_2|}{mgr^2}$$

Solving for $\theta$, we find that

$$\theta = \tan^{-1}\left(\frac{k |q_1| |q_2|}{mgr^2}\right)$$

$$= \tan^{-1}\left[\frac{(8.99\times10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.600\times10^{-6} \text{ C})(0.900\times10^{-6} \text{ C})}{(8.00\times10^{-2} \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m})^2}\right] = 15.4^\circ$$

b. Since $T \cos \theta = mg$, the tension can be obtained as follows:

$$T = \frac{mg}{\cos \theta} = \frac{(8.00\times10^{-2} \text{ kg})(9.80 \text{ m/s}^2)}{\cos 15.4^\circ} = 0.813 \text{ N}$$

39. **REASONING** The drawing shows the arrangement of the three charges. Let $E_q$ represent the electric field at the empty corner due to the $-q$ charge. Furthermore, let $E_1$ and $E_2$ be the electric fields at the empty corner due to charges $+q_1$ and $+q_2$, respectively.

According to the Pythagorean theorem, the distance from the charge $-q$ to the empty corner along the diagonal is given by $\sqrt{(2d)^2 + d^2} = \sqrt{5d^2} = d\sqrt{5}$. The magnitude of each electric field is given by Equation 18.3, $E = k |q|/r^2$. Thus, the magnitudes of each of the electric fields at the empty corner are given as follows:

$$E_q = \frac{k |q|}{r^2} = \frac{k |q|}{(d\sqrt{5})^2} = \frac{k |q|}{5d^2}$$

$$E_1 = \frac{k |q_1|}{(2d)^2} = \frac{k |q_1|}{4d^2} \quad \text{and} \quad E_2 = \frac{k |q_2|}{d^2}$$
The angle $\theta$ that the diagonal makes with the horizontal is $\theta = \tan^{-1}(d / 2d) = 26.57^\circ$. Since the net electric field $E_{\text{net}}$ at the empty corner is zero, the horizontal component of the net field must be zero, and we have

$$E_1 - E_q \cos 26.57^\circ = 0 \quad \text{or} \quad \frac{k|q_1|}{4d^2} - \frac{k|q_1|\cos 26.57^\circ}{5d^2} = 0$$

Similarly, the vertical component of the net field must be zero, and we have

$$E_2 - E_q \sin 26.57^\circ = 0 \quad \text{or} \quad \frac{k|q_2|}{d^2} - \frac{k|q_2|\sin 26.57^\circ}{5d^2} = 0$$

These last two expressions can be solved for the charge magnitudes $|q_1|$ and $|q_2|$.

**SOLUTION** Solving the last two expressions for $|q_1|$ and $|q_2|$, we find that

$$|q_1| = \frac{4}{5} q \cos 26.57^\circ = 0.716 q$$

$$|q_2| = \frac{1}{5} q \sin 26.57^\circ = 0.0895 q$$

76. **CONCEPT QUESTIONS**

a. The drawing at the right shows the electric fields at point P due to the two charges in the case that the second charge is positive. The presence of the second charge causes the magnitude of the net field at P to be twice as great as it is when only the first charge is present. Since both fields have the same direction, the magnitude of $E_2$ must, then, be the same as the magnitude of $E_1$. But the second charge is further away from point P than is the first charge, and more distant charges create weaker fields. To offset the weakness that comes from the greater distance, the second charge must have a greater magnitude than that of the first charge.

b. The drawing at the right shows the electric fields at point P due to the two charges in the case that the second charge is negative. The presence of the second charge causes the magnitude of the net field at P to be twice as great as it is when only the first charge is present. Since the fields now have opposite directions, the magnitude of $E_2$ must be greater than the magnitude of $E_1$. This is necessary so that $E_2$ can offset $E_1$ and still lead to a net field with twice the magnitude as $E_1$. To create this greater field $E_2$, the second charge must now have a greater magnitude than it did in question (a).
a. The magnitudes of the field contributions of each charge are given according to Equation 18.3 as \[ E = \frac{k|q|}{r^2} \]. With \( q_2 \) present, the magnitude of the net field at \( P \) is twice what it is when only \( q_1 \) is present. Using Equation 18.3, we can express this fact as follows:

\[
\frac{k|q_1|}{(2d)^2} + \frac{k|q_2|}{(2d)^2} = 2 \frac{k|q_1|}{d^2} \quad \text{or} \quad \frac{k|q_2|}{(2d)^2} = \frac{k|q_1|}{d^2}
\]

Solving for \(|q_2|\) gives

\[ |q_2| = 4|q_1| = 4(0.50 \, \mu\text{C}) = 2.0 \, \mu\text{C} \]

Thus, the second charge is \( q_2 = +2.0 \, \mu\text{C} \), which is consistent with our answer to Concept Question (a).

b. Now that the second charge is negative, we have

\[
\frac{k|q_2|}{(2d)^2} - \frac{k|q_1|}{d^2} = 2 \frac{k|q_1|}{d^2} \quad \text{or} \quad \frac{k|q_2|}{(2d)^2} = 3 \frac{k|q_1|}{d^2}
\]

Solving for \(|q_2|\) gives

\[ |q_2| = 12|q_1| = 12(0.50 \, \mu\text{C}) = 6.0 \, \mu\text{C} \]

Thus, the second charge is \( q_2 = -6.0 \, \mu\text{C} \), which is consistent with our answer to Concept Question (b).

Chapter 19

9. **SSM WWW REASONING** The only force acting on the moving charge is the conservative electric force. Therefore, the total energy of the charge remains constant. Applying the principle of conservation of energy between locations \( A \) and \( B \), we obtain

\[
\frac{1}{2}mv_A^2 + \text{EPE}_A = \frac{1}{2}mv_B^2 + \text{EPE}_B
\]

Since the charged particle starts from rest, \( v_A = 0 \). The difference in potential energies is related to the difference in potentials by Equation 19.4, \( \text{EPE}_B - \text{EPE}_A = q(V_B - V_A) \). Thus, we have

\[
q(V_A - V_B) = \frac{1}{2}mv_B^2 \tag{1}
\]
Similarly, applying the conservation of energy between locations C and B gives

\[ q(V_C - V_B) = \frac{1}{2} m(2v_B)^2 \]  

(2)

Dividing Equation (1) by Equation (2) yields

\[ \frac{V_A - V_B}{V_C - V_B} = \frac{1}{4} \]

This expression can be solved for \( V_B \).

**SOLUTION**  Solving for \( V_B \), we find that

\[ V_B = \frac{4V_A - V_C}{3} = \frac{4(452 \text{ V}) - 791 \text{ V}}{3} = 339 \text{ V} \]

10. **REASONING** The gravitational and electric forces are conservative forces, so the total energy of the particle remains constant as it moves from point A to point B. Recall from Equation 6.5 that the gravitational potential energy (GPE) of a particle of mass \( m \) is 

\[ \text{GPE} = mgh, \]

where \( h \) is the height of the particle above the earth’s surface. The conservation of energy is written as

\[ \frac{1}{2}mv_A^2 + mgh_A + \text{EPE}_A = \frac{1}{2}mv_B^2 + mgh_B + \text{EPE}_B \]  

(1)

We will use this equation several times to determine the initial speed \( v_A \) of the negatively charged particle.

**SOLUTION** When the negatively charged particle is thrown upward, it attains a maximum height of \( h \). For this particle we have:

\[ v_A = ? \quad v_B = 0 \quad \text{ (at maximum height)} \]

\[ \text{EPE}_A = (-q)V_A \quad \text{EPE}_B = (-q)V_B \]

\[ h_A = 0 \quad \text{ (ground level)} \quad h_B = h \quad \text{ (the maximum height)} \]

Solving the conservation of energy equation, Equation (1), for \( v_A \) and substituting in the data above gives

\[ v_A = \sqrt{\frac{2}{m}[mgh + (-q)(V_B - V_A)]} \]  

(2)

Equation (2) cannot be solved as it stands because the height \( h \) and the potential difference \( (V_B - V_A) \) are not known. We now make use of the fact that a positively charged particle,
when thrown straight upward with an initial speed of 30.0 m/s, also reaches the maximum height \( h \). For this particle we have:

\[
\begin{align*}
  v_A &= 30.0 \text{ m/s} & v_B &= 0 \text{ (at maximum height)} \\
  EPE_A &= (+q) V_A & EPE_B &= (+q) V_B \\
  h_A &= 0 \text{ (ground level)} & h_B &= h
\end{align*}
\]

Solving the conservation of energy equation, Equation (1), for the potential difference \((V_B - V_A)\) and substituting in the data above gives

\[
V_B - V_A = \frac{1}{+q}\left[\frac{1}{2}m(30.0 \text{ m/s})^2 - mgh\right] \tag{3}
\]

Substituting Equation (3) into Equation (2) gives, after some algebraic simplifications,

\[
v_A = \sqrt{4gh - (30.0 \text{ m/s})^2} \tag{4}
\]

Equation (4) cannot be solved because the height \( h \) is still unknown. We now make use of the fact that the uncharged particle, when thrown straight upward with an initial speed of 25.0 m/s, also reaches the maximum height \( h \). For this particle we have:

\[
\begin{align*}
  v_A &= 25.0 \text{ m/s} & v_B &= 0 \text{ (at maximum height)} \\
  EPE_A &= qV_A = 0 \text{ (since } q = 0) & EPE_B &= qV_B = 0 \text{ (since } q = 0) \\
  h_A &= 0 \text{ (ground level)} & h_A &= h
\end{align*}
\]

Solving Equation (1) with this data for the maximum height \( h \) yields

\[
h = \frac{(25.0 \text{ m/s})^2}{2g} = \frac{(25.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 31.9 \text{ m}
\]

Substituting \( h = 31.9 \text{ m} \) into Equation (4) gives \( v_A = 18.7 \text{ m/s} \).

### 24. REASONING AND SOLUTION

a. Let \( d \) be the distance between the charges. The potential at the point \( x_1 = 4.00 \text{ cm} \) to the left of the negative charge is

\[
V = 0 = \frac{kq_1}{d-x_1} - \frac{kq_2}{x_1}
\]

which gives

\[
\frac{q_1}{q_2} = \frac{d}{x_1} - 1 \tag{1}
\]
Similarly, at the point \( x_2 = 7.00 \text{ cm} \) to the right of the negative charge we have

\[
V = 0 = \frac{kq_1}{x_2 + d} - \frac{kq_2}{x_2}
\]

which gives

\[
\frac{q_1}{q_2} = \frac{d}{x_2} + 1 \tag{2}
\]

Equating Equations (1) and (2) and solving for \( d \) gives \( d = 0.187 \text{ m} \).

b. Using the above value for \( d \) in Equation (1) yields \( \frac{q_1}{q_2} = 3.67 \).

---

25. **REASONING AND SOLUTION** The electrical potential energy of the group of charges is

\[
E_{\text{PE}} = \frac{kq_1q_2}{d} + \frac{kq_1q_3}{(2d)} + \frac{kq_2q_3}{d} = 0
\]

so

\[
q_1q_2 + (1/2)q_1q_3 + q_2q_3 = 0
\]

a. If \( q_1 = q_2 = q \), then

\[
q + (1/2)q_3 + q_3 = 0 \quad \text{or} \quad q_3 = -\frac{2}{3}q
\]

b. If \( q_1 = q \) and \( q_2 = -q \) then

\[
-q + (1/2)q_3 - q_3 = 0 \quad \text{or} \quad q_3 = -2q
\]

---

26. **REASONING** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the initial separation of the particles.

**SOLUTION** For two points, \( A \) and \( B \), along the motion, the conservation of energy is

\[
\frac{1}{2}m_1v_{1,A}^2 + \frac{1}{2}m_2v_{2,A}^2 + \frac{kq_1q_2}{r_A} = \frac{1}{2}m_1v_{1,B}^2 + \frac{1}{2}m_2v_{2,B}^2 + \frac{kq_1q_2}{r_B}
\]

Solving this equation for \( 1/r_A \) and setting \( v_{1,A} = v_{2,A} = 0 \) since the particles are initially at rest, we obtain
This equation cannot be solved for the initial separation \( r_A \), because the final speed \( v_{2,B} \) of the second particle is not known. To find this speed, we will use the conservation of linear momentum:

\[
\frac{m_1 v_{1,A}}{r_A} + \frac{m_2 v_{2,A}}{r_B} = \frac{m_1 v_{1,B}}{r_A} + \frac{m_2 v_{2,B}}{r_B}
\]

Setting \( v_{1,A} = v_{2,A} = 0 \) and solving for \( v_{2,B} \) gives

\[
v_{2,B} = -\frac{m_1}{m_2} v_{1,B} = -\frac{3.00 \times 10^{-3} \text{ kg}}{6.00 \times 10^{-3} \text{ kg}} (125 \text{ m/s}) = -62.5 \text{ m/s}
\]

Substituting this value for \( v_{2,B} \) into Equation (1) yields

\[
\frac{1}{r_A} = \frac{1}{0.100 \text{ m}} + \frac{1}{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(8.00 \times 10^{-6} \text{ C}\right)^2} \times \left[\frac{1}{2} \left(3.00 \times 10^{-3} \text{ kg}\right) (125 \text{ m/s})^2 + \frac{1}{2} \left(6.00 \times 10^{-3} \text{ kg}\right) (-62.5 \text{ m/s})^2\right]
\]

\[
\Rightarrow r_A = 1.41 \times 10^{-2} \text{ m}
\]